

# On the Influence of Einstein–Podolsky–Rosen (EPR) Effect on the Probability of Domain-Wall Formation during a Cosmological Phase Transition

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## Abstract

Evading formation of the domain walls in cosmological phase transitions is one of the key problems to be solved for getting agreement with the observed large-scale homogeneity of the Universe. The previous attempts to get around this obstacle led to imposing severe observational constraints on the parameters of the fields involved. Our aim is to show that yet another way to overcome the above problem is accounting for EPR effect. Namely, if the scalar (Higgs) field was presented by a single quantum state at the initial instant of time, then its reduction during a phase transition at some later instant should be correlated even at distances exceeding the local cosmological horizon. By considering a simplest 1D model with  $Z_2$  Higgs field, we demonstrate that EPR effect really can substantially reduce the probability of spontaneous creation of the domain walls.

## 1 Introduction

The problem of domain-wall formation during spontaneous breaking of discrete symmetry was emphasized for the first time by Zel'dovich, Kobzarev, and Okun as early as 1974–1975 [1], i.e., just when the role of Higgs fields in cosmology began to be recognized. This problem

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is associated with the fact that the observed region of the Universe contains a great number of domains that were not causally-connected at the instant of phase transition, and the stable vacuum states of such domains after the symmetry breaking will differ from each other. As a result, a network of domain walls, involving considerable energy density, should be formed. But, on the other hand, the presence of such domain walls is incompatible with the observed homogeneity of the Universe.

The previously-undertaken attempts to resolve the above problem were based on imposing severe constraints on the parameters or modifying the field theories involved (see, for example, [2]).

Aim of the present report is to show that, in principle, the domain-wall problem may be resolved by a natural way if we take into account the fact that Higgs condensate represents a single quantum state, which should experience Einstein–Podolsky–Rosen (EPR) correlations [3] during its reduction to the state of broken symmetry.

This point of view seems to be especially attractive due to the recent experimental achievements, such as (a) the quantum-optical experiments, which confirmed a presence of EPR correlations of the single photon states at considerable distances ( $\sim 10$  km), and (b) the studies of Bose–Einstein condensate of ultracooled gases, which demonstrated that all predictions of the “orthodox” quantum mechanics are valid for a single quantum state involving even a macroscopic amount of substance.

So, if we believe that EPR correlations really take place in Higgs condensate, then it should be expected that the probabilities of various realizations of the Higgs-field configurations after the symmetry breaking will be distributed by Gibbs law. As a result, the high energy concentrated in the domain walls (and, therefore, contradicting the astronomical observations) turns out to be just the reason why probability of the respective configurations is strongly suppressed.

The basic question arising here concerns the efficiency of such suppression for a particular set of parameters of the Higgs field under consideration. The same question can be reformulated by an opposite way: What parameters should the Higgs field (and its phase transition) have for the probability of domain-wall formation to be substantially reduced? We shall try to give a quantitative answer to this question in the next section.

## 2 The Model of Phase Transition Allowing for EPR Correlations

Let us consider the simplest one-dimensional cosmological model with metric

$$ds^2 = dt^2 - a^2(t) dx^2. \quad (1)$$

By introducing the conformal time  $\eta = \int dt/a(t)$ , expression (1) can be reduced to the form  $ds^2 = a^2(t) \{d\eta^2 - dx^2\}$ ; so that the light rays ( $ds^2 = 0$ ) are described as  $x = \pm \eta + \text{const}$  [4].

As a result, the observed region of the Universe will contain  $N$  domains that were causally-disconnected at the instant of phase transition:

$$N = (\eta - \eta_0)/\eta_0 \approx \eta/\eta_0, \quad (2)$$

where  $\eta_0$  is the conformal time corresponding to the phase transition.

The Higgs field  $\varphi$ , possessing the symmetry group  $Z_2$ , after reduction to the state of broken symmetry can be in one of two stable vacua ( $+\varphi_0$  or  $-\varphi_0$ ) in each of the above-mentioned domains. The energy of a “wall” between two domains with different vacua will be denoted by  $E$ . Besides, for the sake of self-consistency, periodic boundary conditions will be imposed at the opposite sides of the observable region; so that the possible total number of the domain walls is always even.

So, under assumptions formulated above, the probability of realization of the Higgs-field configuration involving  $2k$  domain walls equals

$$P_N^{2k} = 2 A_N \frac{N!}{(2k)! (N-2k)!} e^{-2kE/T}, \quad (3)$$

where  $T$  is the temperature of phase transition, and  $A_N$  is the normalization factor, defined as

$$A_N^{-1} = \sum_{k=0}^{[N/2]} \frac{2 N!}{(2k)! (N-2k)!} e^{-2kE/T}, \quad (4)$$

where square brackets in the upper limit of the sum denote the integer part of a number.

Normalization factor (4) can be easily calculated in two limiting cases: (a) when neighboring terms of the sum differ from each other only slightly, or (b) when main contribution to the sum is done by a few first terms.

As regards the first case, summation can be approximately extended to the terms with any number of the domain walls (both odd and even) and finally gives

$$A_N^{-1} \approx \left(1 + e^{-E/T}\right)^N; \quad (5)$$

so that

$$P_N^{2k} \approx \frac{2 N!}{(2k)! (N-2k)!} \frac{e^{-2kE/T}}{(1 + e^{-E/T})^N}. \quad (6)$$

In particular, probability of the Higgs-field configuration without any domain walls (which is just the case actually observed) equals

$$P_N^0 \approx \frac{2}{(1 + e^{-E/T})^N}. \quad (7)$$

It is interesting to find restriction on the domain-wall energy  $E$  and phase-transition temperature  $T$  for the probability of absence of the domain walls to be greater than or equal to some specified number  $p$  ( $0 < p < 1$ , for example,  $p = 1/2$ ):

$$E/T \geq \ln \frac{1}{(2/p)^{1/N} - 1} \approx \ln N - \ln \ln (2/p) \approx \ln N. \quad (8)$$

As regards the opposite case, when normalization factor (4) is determined by the terms with small  $k$ , it can be estimated by taking into account only the first two terms:

$$A_N^{-1} \approx 2 \left\{ 1 + \frac{1}{2} N^2 e^{-2E/T} \right\}; \quad (9)$$

and the respective probability of the Higgs-field configuration involving  $2k$  domain walls is

$$P_N^{2k} \approx \left\{ 1 - \frac{1}{2} N^2 e^{-2E/T} \right\} \frac{N!}{(2k)! (N-2k)!} e^{-2kE/T}. \quad (10)$$

Then, the probability that domain walls are absent at all is

$$P_N^0 \approx 1 - \frac{1}{2} N^2 e^{-2E/T}, \quad (11)$$

and it will be greater than or equal to the specified number  $p$  if

$$E/T \geq \ln N - \frac{1}{2} \ln (2(1-p)) \approx \ln N, \quad (12)$$

i.e., to a first approximation, it is again determined by the logarithmic function of  $N$ .

### 3 Conclusion

There is no doubt that a considerably more sophisticated analysis must be carried out to draw unambiguous conclusion on the role of EPR correlations in the phase transitions of Higgs fields. Nevertheless, our calculations, based on the simplest model, showed that such possibility is quite promising. Since, as follows from (8) and (12), the ratio  $E/T$  differs from unity only by  $\ln N$ , then the required temperature of phase transition is of the same order of magnitude as the domain-wall energy even at a large number of the causally-disconnected regions  $N$ . Therefore, the required parameters of the phase transition could be satisfied in some particular kind of the field theory.

### References

- [1] Ya.B. Zel'dovich, I.Yu. Kobzarev, and L.B. Okun. "Cosmological Consequences of a Spontaneous Breakdown of a Discrete Symmetry." *Zh. Eksper. Teor. Fiz. (in Russian)*, vol. **67**, no. 1(7), pp. 3–11 (1974). [Translated in *Sov. Phys. JETP*, vol. **40**, p. 1 (1975).]
- [2] S.E. Larsson, S. Sarkar, and P.L. White. "Evading the Cosmological Domain Wall Problem." *Phys. Rev. D*, vol. **55**, no. 8, pp. 5129–5135 (1997).
- [3] A. Einstein, B. Podolsky, and N. Rosen. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Phys. Rev.*, vol. **47**, pp. 777–780 (1935).
- [4] C.W. Misner. "Mixmaster Universe." *Phys. Rev. Lett.*, vol. **22**, no. 20, pp. 1071–1074 (1969).